Skewness and market investors risk preferences

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Abstract
Risk preferences are generally measured with utility functions, which are subjective and usually restrict market agents to be risk averse. The current study investigates investors’ preferences toward risk from the index price movement perspective. In a risk neutral market, future prices will increase or decrease with the same probability. However, if the market representative-agents are risk averse, the probabilities of future price increase and decrease will be different and can easily be detected through the conditional skewness. We investigated the risk preferences for the American and Japanese markets. Although the American S&P 500 index shows historical risk neutrality over the past 10 years, everyday movements show negative skewness and thus risk aversion; the Japanese Nikkei 225 has negative conditional skewness and thus Japanese investors are risk averse.

Keywords: generalized t-distribution, risk preferences, skewness, stock index, market agent.
JEL classification: C13 ; C16 ; C22 ; G11 ; G12

1. Introduction
Investors’ risk preferences are usually investigated through the utility function, which can be quadratic or cubic (Post et al., 2008). Risk aversion is then inferred from this subjective function. However, risk aversion depends greatly on the predefined subjective utility function, which, in most cases follows the usual hypothesis of risk aversion of the investors assumed by the majority of researchers.
Recently, some scholars tried to investigate risk aversion through a different method based on the true market behavior observed from price movements, e.g., Kang & Kim (2007) used the subjective probability function derived from the pricing kernel implied by option prices of the S&P 500 options and found evidence of risk neutrality.
Options prices are used in recent studies (Bliss & Panigirtzoglou, 2004; Kang & Kim, 2006) to estimate the implied probability densities functions (PDFs) of the prices of the underlying assets. These PDFs are future forecasts of the distribution of prices, and they are much more responsive to changing market expectation than distribution derived from historical data. However, these PDFs are risk neutral and cannot correspond to a representative market investor whose risk preferences are unknown. If the investors are rational, their subjective distribution of future prices should correspond on average to the real world distribution. Thus, the degree of risk can be inferred from the difference between the true distribution of future values of an asset and the risk neural distribution.
Another behavior of market investors is the tendency to be more averse toward risk when stock prices fall than when stock prices rise. This is known as the leverage effect, for e.g., a firm with outstanding equity typically becomes more highly leveraged when the value of the firm falls; this raises equity returns volatility if the returns on the firm as a whole are constant. Additionally, market investors are more sensitive to bad news (negative shocks) than to good news (positive shocks), so the conditional risk as represented by price volatility tends to increase more than expected when past returns show a negative behavior. However, this tendency to be more averse toward risk just after the bad news could be misleading to understand the market investor’s attitude at the time when he’s about to make the decision to buy or sell equity.
The relevant issue becomes: How can we investigate risk aversion from the sole observation of expected index price movements, and without applying a subjective utility function?
Two key factors characterize market investors risk preferences: first, the degree of risk aversion of a representative agent (or market) is time-variant, and second, risk aversion is deduced from the future distribution of asset prices forecasts.
The remaining of the paper will be divided into three sections: In section 2 our methodology will be presented, section 3
will be devoted to the data and the model estimation, and finally we conclude in section 4.

2. Methodology

Risk preferences of investors are not necessarily constant over time, as argued by Barberis et al. (2001) and Chue (2002); they are subject to wealth and consumption fluctuations, and to prior investment performances. Moreover, the degree of risk is a measure, at a given time, of future expectations of representative market investors.

The current price index is the result of a confrontation of all market agents buy / sell orders at the specific moment. These prices are generated by the endogenous agents’ perceptions, so the market will depend on the agents forecasting rules and the associated learning systems. In a market where agents are risk indifferent, future prices will have the same probability of increasing or decreasing. Risk aversion will be characterized by the unbalanced future probability of price movements, i.e., the probability that a future price decreases is different from the probability that the same price increases. Negative skewness indicates that large negative changes are more likely to happen than large positive changes, and market agents are expecting these changes through the future distribution of their best forecasts, which indicates their risk aversion.

There is a need to estimate the time-variant probability density function of future index prices or conditional distribution, and compare the probability that the index price increases to the probability that the index price decreases. A much known model, which can offer the estimation of time-varying price movement, is the GARCH model, which, combined with a non-symmetric flexible distribution can provide a powerful tool to estimate the conditional future distribution of actual prices and the conditional skewness. Conditional skewness can be a good measure of market agents’ risk preferences.

The model is a simple GARCH(1,1) with a Skewed Generalized T-distribution. Although non-symmetric models like EGARCH and GJR-GARCH allow for leverage effect, it is more plausible to capture the risk preference independently of past index, so the entire conditional risk will be shown in the conditional skewness of the newly developed distribution.

The GARCH(1, 1) model is given by:

\[
\begin{align*}
    r_t &= c + e_t \sqrt{h_t}; \quad e_t \rightarrow \text{IID}(0,1) \\
    h_t &= \alpha_0 + \alpha_t r_{t-1}^2 + \beta_t h_{t-1}
\end{align*}
\]

(Eq. 1)

\(r_t\) is the index return, \(c\) is its mean, and \(e_t\) is an Independently and Identically Distributed random variable with zero mean and unit variance, and which follows a Skewed Generalized T-distribution (SGT). Following BenSaïda (2007), Refining the distribution of GARCH models: application to stock indexes returns. Social Science Research Network, http://ssrn.com/abstract=980042, an extension to the generalized t-distribution is the introduction of an asymmetric parameter \(\lambda\). The distribution becomes:

\[
f(x) = \frac{\eta}{2^\theta \cdot B\left(\frac{1}{\eta}, \frac{k}{\eta}\right)} \left[1 + \frac{|x+\mu|^{-\eta}}{\left(1 + \text{Sign}(x+\mu)\lambda\right)\theta'} \right]^{-\frac{k+1}{\eta}}
\]

(Eq. 2)

Under the conditions that \(\eta > 0, k > 2, |\lambda| < 1, \text{ and } \theta > 0.\) The expectation \(E(x)\) and variance \(\text{Var}(x)\) must both equal 0 and 1 respectively, hence:
\[
\mu = 2\delta \lambda \frac{\Gamma \left( \frac{2}{\eta} \right) \Gamma \left( \frac{k-1}{\eta} \right)}{\Gamma \left( \frac{1}{\eta} \right) \Gamma \left( \frac{k}{\eta} \right)}
\]
\[
\theta = \frac{\Gamma \left( \frac{1}{\eta} \right) \Gamma \left( \frac{k}{\eta} \right)}{\left[ (1+3\lambda^2) \Gamma \left( \frac{1}{\eta} \right) \Gamma \left( \frac{3}{\eta} \right) \Gamma \left( \frac{k}{\eta} \right) \Gamma \left( \frac{k-2}{\eta} \right) - 4\lambda^2 \Gamma \left( \frac{2}{\eta} \right)^2 \Gamma \left( \frac{k-1}{\eta} \right)^2 \right]^{\frac{1}{2}}}
\]

- Setting \( \lambda = 0 \), this distribution becomes symmetric;
- For \( \lambda > 0 \), the SGT is skewed to the right, large positive changes are more likely to occur;
- For \( \lambda < 0 \), the SGT is skewed to the left, large negative changes are more likely to occur.

The skewness of this distribution is:
\[
\text{Skewness} = 4\theta^3 \lambda \left( 1 + \lambda^2 \right) \frac{\Gamma \left( \frac{4}{\eta} \right) \Gamma \left( \frac{k-3}{\eta} \right)}{\Gamma \left( \frac{1}{\eta} \right) \Gamma \left( \frac{k}{\eta} \right)} - 3\mu - \mu^3 \quad \text{(Eq. 3)}
\]

The kurtosis is:
\[
\text{Kurtosis} = \theta^4 \left( 1 + 5\lambda^2 \left( 2 + \lambda^2 \right) \right) \frac{\Gamma \left( \frac{5}{\eta} \right) \Gamma \left( \frac{k-4}{\eta} \right)}{\Gamma \left( \frac{1}{\eta} \right) \Gamma \left( \frac{k}{\eta} \right)} - 4\mu \cdot \text{Skewness} - 6\mu^2 - \mu^4 \quad \text{(Eq. 4)}
\]

The probability of negative returns is different from the probability of positive returns due to the asymmetry effect:
\[
P[x < 0] = P[x < -\mu] + P[-\mu < x < 0] = \frac{1-\lambda}{2} + \int_{-\mu}^{0} f(x) dx
\]
\[
P[x > 0] = P[x > -\mu] - P[-\mu < x < 0] = \frac{1+\lambda}{2} - \int_{-\mu}^{0} f(x) dx
\]

Proofs are in the Appendix.

The symmetric generalized t-distribution can nest a large variety of other distributions such as the standard t-distribution when \( \eta = 2 \), the degree of freedom is \( k \); the Cauchy distribution when \( \eta = 2 \) and \( k = 1 \); the Generalized Error Distribution (also known as power exponential distribution) when \( k = \infty \), the degree of freedom is \( \eta \); the Laplace distribution when \( \eta = 1 \) and \( k = \infty \); the standard normal distribution when \( \eta = 2 \) and \( k = \infty \); and the uniform distribution when \( \eta = \infty \) and \( k = \infty \). Adding asymmetry parameter \( \lambda \) will enhance this distribution to better approximate the dispersion of the future price index.

Risk preferences depend on the sign of the asymmetry parameter \( \lambda \). Indeed, when \( \lambda = 0 \), the return has the same probability to be either positive or negative (index price increases or decreases with the same probability), market representative investors are risk neutral. On the other hand, when \( \lambda \) is negative, the probability of large negative returns (price index decreases) is greater than the probability of large positive returns (price index increases); in this case market representative investors are risk averse, and vice versa.
3. Data and Estimation

Investor preferences will be studied for the two major markets, which are the American and Japanese stock exchanges. Data are the daily S&P 500 and the Nikkei 225 index closing rate from December 1, 1998 until November 30, 2007. For risk neutral investors, their expectations for future changes will be conditioned by a symmetric distribution, so the chances that future prices increase or decrease are the same. Figure 1 shows a risk neutral market where the expected index increases and decreases with the same probability.

The relevant variable is the return of the stock index, computed as the logarithmic difference between two successive index rates. Thus, when the price index increases (decreases), the return is positive (negative). The descriptive statistics are shown in Table 1.

The descriptive statistics show that the mean of the two returns are statistically not different from zero, the coefficient $c$ can then be omitted from the model (Eq. 1). The unconditional kurtoses of both returns clearly show that the distribution is far from normal, justifying the choice of the generalized $t$-distribution.

It is important to note that the unconditional skewness of the S&P 500 return is statistically not different from zero; we can conclude that the investors have the same expectation of an increase (positive return) and of a decrease (negative return). However, the reported skewness in Table 1 is unconditional, i.e., it corresponds to the studied period as a whole and not at every moment of the investment horizon. Consequently, a null unconditional skewness does not indicate risk neutrality of the investors; instead, risk aversion is identified by the conditional skewness.

Results for the S&P 500 are presented in Table 2.

Results for the Nikkei 225 are presented in Table 3.

The $p$-value of the null hypothesis that the conditional skewness equals zero is computed with the delta method, which can give the variance of any linear or non-linear combination of the estimated coefficients.

For both stock markets, American and Japanese, conditional skewnesses are negative and different from zero, which support the hypothesis of risk aversion of their market agents.

4. Conclusion

We investigated risk preferences for the American and Japanese markets with the analysis of the expectations of the representative market agents toward future index prices. Risk aversion is determined from the asymmetric distribution perspective.

Although the American index S&P 500 shows historical risk neutrality over the past 10 years, everyday movements show negative skewness and thus risk aversion; the Japanese Nikkei 225 has negative conditional skewness and thus indicates the risk aversion of Japanese investors.

<table>
<thead>
<tr>
<th>Stock Index Return</th>
<th>S&amp;P 500</th>
<th>Nikkei 225</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.00010221</td>
<td>2.50506E-05</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.00023519</td>
<td>0.000291437</td>
</tr>
<tr>
<td>$p$-value (Mean = 0)</td>
<td>0.6639</td>
<td>0.9315</td>
</tr>
<tr>
<td>Sample Variance</td>
<td>0.00012517</td>
<td>0.000187877</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>5.21874421</td>
<td>4.676934146</td>
</tr>
<tr>
<td>$p$-value (Kurtosis = 3)</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.05330434</td>
<td>-0.13778213</td>
</tr>
<tr>
<td>$p$-value (Skewness = 0)</td>
<td>0.2996</td>
<td>0.0083</td>
</tr>
<tr>
<td>Range</td>
<td>0.11578946</td>
<td>0.144557279</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.0600451</td>
<td>-0.07233984</td>
</tr>
</tbody>
</table>
Table 2: Results for the S&P 500 return

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>5.746e-07</td>
<td>2.5339e-07</td>
<td>2.2676</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.061871</td>
<td>0.0098261</td>
<td>6.296</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.93531</td>
<td>0.0099644</td>
<td>93.864</td>
</tr>
<tr>
<td>$\eta$</td>
<td>1.7004</td>
<td>0.075489</td>
<td>22.5246</td>
</tr>
<tr>
<td>$k$</td>
<td>21.33</td>
<td>0.00075669</td>
<td>28188</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>-0.083109</td>
<td>0.028051</td>
<td>-2.9628</td>
</tr>
</tbody>
</table>

Log-likelihood: 7254.5673
Akaike Info. Criterion: -14497.1348
Num. of Coefs: 6
Schwarz Info. Criterion: -14462.7881
Conditional Skewness: -0.1986
Conditional Kurtosis: 3.8684
p-value (Cond. Skew = 0): 0.0026

Table 3: Results for the Nikkei 25 return

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>2.0305e-06</td>
<td>8.4708e-07</td>
<td>2.3971</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.066447</td>
<td>0.01132</td>
<td>5.8700</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.92486</td>
<td>0.013062</td>
<td>70.8049</td>
</tr>
<tr>
<td>$\eta$</td>
<td>1.747</td>
<td>0.17067</td>
<td>10.2363</td>
</tr>
<tr>
<td>$k$</td>
<td>17.47</td>
<td>10.154</td>
<td>1.7205</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>-0.056524</td>
<td>0.028793</td>
<td>-1.9631</td>
</tr>
</tbody>
</table>

Log-likelihood: 6490.8933
Akaike Info. Criterion: -12969.7866
Num. of Coefs: 6
Schwarz Info. Criterion: -12935.5767
Conditional Skewness: -0.1361
Conditional Kurtosis: 3.8848
p-value (Cond. Skew = 0): 0.0472

References
Appendix

Following Gradshteyn & Ryzhik (2007), p. 341, § 3.241.4,

\[
\int \frac{x^p}{(1 + a \cdot x^n)^k} \, dx = \frac{\Gamma \left( \frac{p + 1}{n} \right) \Gamma \left( \frac{p - k \cdot n + 1}{n} \right)}{n \cdot a \cdot \Gamma(k)}
\]

under the condition that \(n > 0, p > -1\), and \((k \cdot n - p) > 1\).